Analysis of Gas-Solid Particle Flows in Shock Tubes

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Theme

THE use of shock tubes for experimental investigations of high-speed suspension flows has motivated the development of a corresponding numerical calculation procedure. The Particle-In-Cell (PIC) method[1] has been adapted to handle suspension flows by treating each phase as a set of discrete "mass points." As in the single-phase PIC calculation the flowfield is divided into small regions fixed in space (cells), but in the modified procedure phase interactions are incorporated into the equations for the changes in the properties of the materials occupying the cells. Details of the calculation are given in Ref. 2.

The purpose of this study was to investigate the dependence of the calculated results on the assumed phase interaction (drag and heat transfer).

The behavior of suspension flows in shock tubes also was investigated for the limiting case of problem times that are large compared with the characteristic velocity and thermal equilibration times. In this case, there exists a zone behind the shock within which the phases are in velocity and thermal equilibrium. These equilibrium properties, as well as the equilibrium shock speed, can be calculated by applying the shock tube equations to an "equivalent gas"[3] defined in terms of the properties of the suspension, thereby providing a check on the calculated flow properties.

Contents

Equilibrium flow in shock tubes: In the equilibrium shock tube calculations, an initial pressure ratio of 5:1 was used to simulate typical experimental determinations of drag coefficients.[4] The drag interaction was calculated from the empirical drag coefficient, particle Reynolds number relation of Ref. 5

$$C_D = 0.48 + 28.0 Re^{-0.85} \tag{1}$$

while an expression from Ref. 6 provided the heat-transfer rate

$$Nu = 2.0 + 0.6 Re^{1/2} Pr^{1/3} \tag{2}$$

Figure 1 shows the flow parameters at 2.4 msec calculated by the PIC method, with the thermal equilibration time τ_T deliberately reduced to 10^{-4} sec, or one tenth the value calculated for the velocity equilibration time τ_V, in order to maintain the phases near thermal equilibrium at every location in the shock tube. Also shown are the values predicted by the shock tube equations using the equivalent specific heat ratio, γ_E, and sound speed, C_E, calculated as in Ref. 3

$$\gamma_E = \gamma_o(1+\eta\delta)(1+\gamma_o\eta\delta)^{-1} \tag{3}$$

$$C_E = C_o(1+\eta\delta)^{1/2}[(1+\eta)(1+\gamma_o\eta\delta)]^{-1/2} \tag{4}$$

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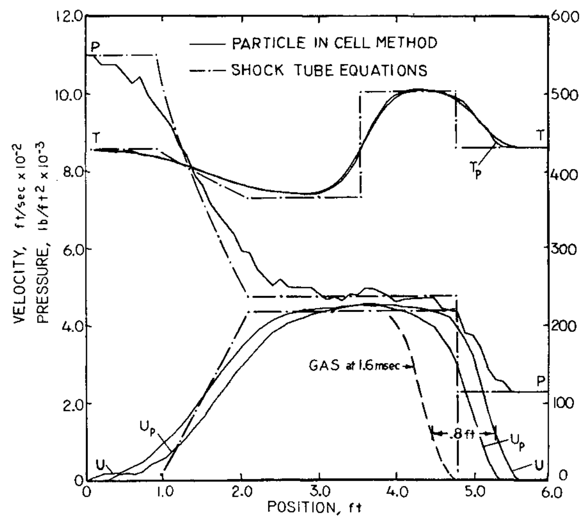

Fig. 1 Pressure, velocity, and temperature profiles for a shock tube containing a suspension of spherical particles in air, according to the particle in cell method and the shock tube equations. Initial conditions: pressure ratio = 5.0, $\eta = 1.0$, $\gamma = 1.4$, $\delta = 0.5$, $C_o = 1000$ fps, $\tau_T = 1.0 \times 10^{-4}$ sec, $t = 2.4$ msec.

where η and δ are mass-loading and particle/fluid-specific-heat ratios, respectively, and C_o is the frozen sound speed associated with the gas alone. Using the values $\eta = 1.0$, $\delta = 0.5$, and $\gamma_o = 1.4$ in Eqs. (3) and (4), leads to the equivalent gas properties, $\gamma_E = 1.235$ and $C_E = 665$ fps. Figure 1 shows the agreement of the PIC flow properties behind the shock with the equivalent gas values. Furthermore, the distance between the shock fronts in the gas at 1.6 msec and 2.4 msec is 0.8 ft, which shows that the shock speed is within 5% of the predicted value of 950 fps.

Calculations using a mass loading ratio of 10.0 serve to illustrate phenomena associated with highly loaded suspensions. Figure 2 shows that the resulting flow is nearly isothermal and, in fact, the polytropic exponent [calculated from Eq. (3)] used in the shock tube equations is 1.05. The particle volume fraction of 1% was neglected in calculating the flow properties from the shock tube equations based on the work presented in Ref. 7. Because of the low shock speed in this highly loaded suspension (382 fps), the length of the "equilibrium" region behind the shock is small during the problem time considered (2.4 msec). Despite the small number of cells occupied by the shock, contact discontinuity, and rarefaction wave, the equilibrium values agree closely with the predictions of the shock tube equations. However, the results suggest using a large number of cells to represent these regions accurately.

Experimental studies of suspensions commonly employ a particle-free driver—generally air at the temperature of the suspension which fills the lower pressure end of the shock tube. This situation is illustrated in Fig. 3. The suspension which initially filled the low pressure end of the shock tube had a mass loading ratio of 10.0 and $\tau_T = \tau_V = 1.0 \times 10^{-3}$ sec. In this case the rarefaction wave traveled considerably faster than the

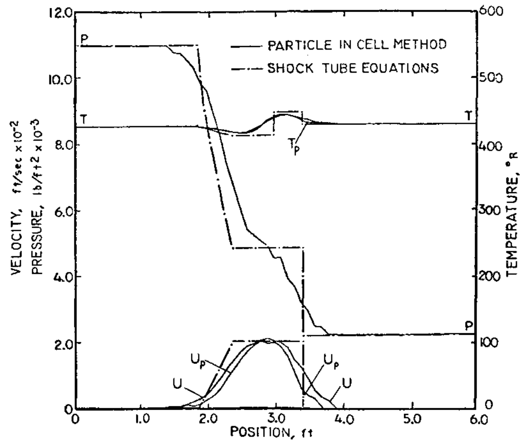

Fig. 2 Pressure, velocity, and temperature profiles for a shock tube containing a suspension, according to the particle in cell method and the shock tube equations, $\eta = 10.0$, $\tau_T = 1.0 \times 10^{-3}$ sec. All other conditions are the same as for Fig. 1.

shock, and reflection has occurred at the end wall of the high-pressure section. These results indicate that shock tube experiments involving such flows require a relatively long high pressure section.

Drag and heat-transfer effects on shock tube transients: Transient shock tube behavior was investigated by choosing problem times which are an order of magnitude shorter than the characteristic times. These calculations were performed using a time increment of 5×10^{-7} sec and a cell length of 5×10^{-3} ft; the results were verified by repeating some of the calculations with the time increment and cell length each halved. The initial suspension properties were identical to those used to obtain Fig. 3, and four combinations of drag and heat-transfer laws were considered as listed in Table 1.

Table 1 Drag and heat interactions for transient shock tube studies with $\eta = 10$

Drag law	Heat-transfer law	Figure
Equation (1)	Equation (2)	4a
	$Nu = 2$	4b
$C_D = 24/Re$	Equation (2)	4c
(Stokes law)	$Nu = 2$	4d

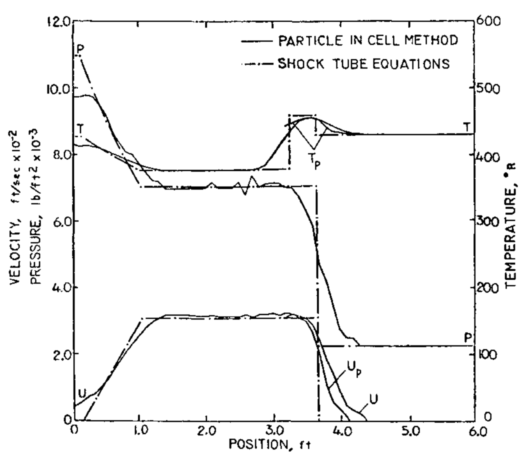

Fig. 3 Pressure, velocity, and temperature profiles for a shock tube with a suspension in the low pressure end only, according to the particle in cell method and the shock tube equations. All other conditions are the same as for Fig. 2.

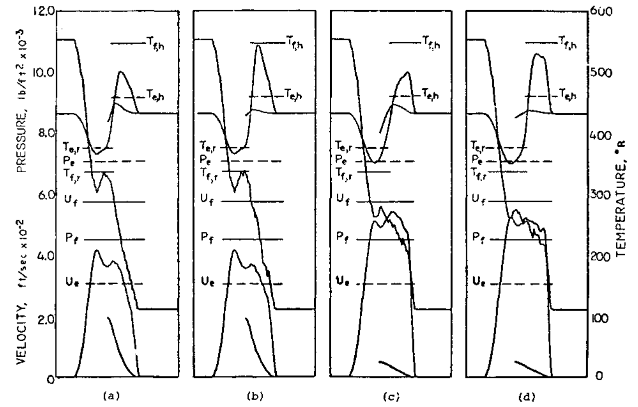

Fig. 4 Influence of assumed drag and heat-transfer laws on the transient behavior of a shock tube containing a suspension in the low pressure end only (see Table 1). $t = 1.0 \times 10^{-4}$ sec. All other conditions are the same as for Fig. 3.

Property profiles at 10^{-4} sec calculated with the empirical drag and heat-transfer laws are shown in Fig. 4a, in which frozen and equilibrium flow properties have also been indicated. At the problem time illustrated, the gas properties would be expected to be near their frozen values U_f, P_f, $T_{f,h}$, and $T_{f,r}$ (the subscripts h and r refer to the shock-heated and rarefaction zones, respectively); yet the gas has departed significantly from a frozen state. Furthermore, those particles which have been subjected to the high velocity gas during the entire time preceding that illustrated, have attained a velocity which is about half of the equilibrium velocity. The temperatures of these particles remain low as a result of their being subjected first to the shock-heated gas and then to the expanded gas.

Figures 4a and 4b show that the heat-transfer law has no effect on the pressure or velocity profiles, or on the temperature of the rarefied gas, and affects only the temperatures in the shock-heated region. Comparisons of Figs. 4a and 4c, and of Figs. 4b and 4d, show that the particulate drag can affect the temperature by influencing the shock speed during the transient portion of the flow. However, this phenomenon was not noticeable at the lower loading, $\eta \doteq 1.0$. In short, there is no evident dependence of the velocity and pressure on the heat-transfer law, but a weak dependence of the temperature in the shock-heated region on the drag law becomes evident at high mass loading ratios. In an earlier investigation of the steady flow of a suspension with low loading ($\eta = 0.2$) across a normal shock,[3] it was observed that the gas velocity and pressure depend primarily on the drag law. The present work shows that this observation applies as well to the unsteady flow of suspensions over a wide range of loading.

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